

*How to deal with Janus' face of natural numbers<sup>1</sup>*  
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### **1. The dilemma**

Arithmetical expressions behave syntactically in two different ways, either as singular terms in identity statements or as predicates of concepts in adjectival statements. To take an example from *Grundlagen* §57, “the number of Jupiter’s moons is 4” (1) is an identity statement in which, the arithmetical expressions behave as singular terms. The verb ‘is’ is not a copula here, but it means ‘is identical to’. A different way to write down (1) is “Jupiter has 4 moons” or “There are 4 moons of Jupiter” (2) where the arithmetical expression ‘4’ behaves as an adjective.

The two forms of syntactical behaviour of arithmetical terms give rise to opposite accounts of the ontological status of natural numbers. The form ‘ $\forall x:Fx = n$ ’ (we will call it the *substantival* form) ordinarily supports the view that natural numbers are *objects* and that arithmetical terms are their names. The form ‘ $\exists_n x Fx$ ’ (we will call it the *predicative* form) is usually connected with accounts of natural numbers as *properties*, either of physical collections or of sortal concepts. So, a dilemma arises for those philosophers who take under consideration linguistic issues in order to support their metaphysical claims. Crispin Wright has described this dilemma in the following passage:

“...the choices are just two: either (following Frege) we take the basic form of numerical expression to be a singular term, definable by reference to an operator on concepts and re-parse ordinary adjectival statements of number as statements of identity; Or we take the basic form of numerical expression to be a predicate of concepts, i.e. a quantifier, and seek to re-parse the apparently substantival uses of numerical vocabulary with which number theory abounds”. (Wright, 1983, p. 36)

This paper takes under consideration both forms of numerical expressions and their respective metaphysical interpretations and sketches the difficulties of each. In particular, it investigates the relation between the substantival and the predicative form and suggests that among the two there is no prevalent. Then it articulates a proposal according to which the substantival form is equivalent to the predicative form by presenting an appropriate equivalence principle. It concludes with suggestions about a possible account of natural numbers as both objects and universals.

### **2. Natural numbers as objects**

On the Fregean view, the individual instances of the concept *natural number*, namely the numbers 0, 1, 2, 3, ... are taken to be *abstract objects* which usually is taken to mean *abstract particulars*. In *Grundlagen* §57, Frege suggests that the adjectival sentence “Jupiter has 4 moons” *should* be re-parsed as an identity statement: “The

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number of Jupiter's moons is 4". In the latter context, the arithmetical expression ceases to play an adjectival role. Frege believes that this form of arithmetical statements reveals the status of natural numbers as self-subsistent objects.

The neo-Fregeans (Wright 1983, pp. 13-14), (Hale & Wright, 2001, p. 48) appeal to the substantival form ' $Nx:Fx = n$ ' too and they claim that a proper argument in favour of mathematical realism should be based on it. According to their argument, arithmetical expressions function in certain atomic sentences as singular terms. The sentences in which they function so are true. So, there are objects to which those expressions refer. On the basis of the neo-Fregean argument, natural numbers are the referents of singular arithmetical terms which occur in appropriate true sentences. Thus, the problem about the status of natural numbers is reduced to questions about the truth of certain arithmetical identities and the syntactical function of arithmetical expressions.

The first presupposition of the neo-Fregean claim is that we can fix truth conditions for arithmetical identities of the type " $Nx:Fx = Nx:Gx$ ", that is, we are able to know whether the identity statement "*the number of the concept F is identical to the number of the concept G*" is true. This requirement is met by means of Hume's Principle:

$$(N \equiv) \quad (\forall F)(\forall G) [(Nx:Fx = Nx:Gx) \leftrightarrow (Fs I-I Gs)]$$

which provides truth conditions for arithmetical identities and can perfectly be considered as a criterion of identity for natural numbers.

The second presupposition of the neo-Fregean claim is that we possess certain criteria by which we can characterize arithmetical terms as singular terms. Frege, for example, stressed that arithmetical terms usually occur in identity statements, they come after the definite article 'the' and they get positions of the logical subjects of arithmetical sentences. Neo-Fregeans, however, attempted to formulate strict syntactical criteria which aim to discriminate singular terms from other expressions.

The neo-Fregean's criteria for singular termhood include: Dummett's (1973, pp. 59-60) inferential criteria, the negation asymmetry criterion and some extra supplementary tests<sup>2</sup>. To cut a long story short, the neo-Fregean proposal aims at ruling out quantifier words, indefinite noun phrases, predicates and relational expressions as well as other phrases which express generality but they often stand in places where singular terms can go. So, their first claim is that singular terms can be recognized among several kinds of expressions. Secondly, they hold that since arithmetical expressions pass successfully the criteria in question then they must be characterized as singular terms. Hence, on this view, singular termhood can be secured for arithmetical expressions.

The neo-Fregean claim that natural numbers are objects has received a fair amount of attention for many years. Yet it has also received criticism, especially

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2. Firstly, if 't' functions as a singular term in a sentence then it must necessarily satisfy the following conditions:

- I. From any sentence "A(t)" it shall be possible to infer "there is something such that A(it)".
- II. From two sentences "A(t) and B(t)" it shall be possible to infer "there is something such that A(it) and B(it)".
- III. A disjunction "A(t) or B(t)" may be inferred from "it is true of t that A(it) or B(it)".

Secondly, the "negation asymmetry" criterion is based on Aristotle's view that qualities have negations while primary substances don't. According to that criterion, singular terms are distinguishable from predicates because the latter have contradictories while the former don't. Additionally, auxiliary tests are also offered in order to guarantee correct application of all tests and especially the negation asymmetry criterion on a variety of cases. Of course, this is a rough presentation of the neo-Fregean tests. Technical details about them are discussed in (Hale & Wright, 2001, pp. 31- 47, 49-70).

concerning the syntactical status of numerical expressions in arithmetical language, as well as the degree of efficiency of the proposed criteria mentioned above. The attempt to discriminate singular terms from other types of expressions has been criticized mainly on the grounds that the proposed tests do not always hit their target, since they don't succeed in excluding plenty of troublesome and misleading expressions which apparently behave as singular terms. Szabó (2003, pp. 38-39) correctly notes that expressions like 'the existence of a proof', 'the occurrence of a symptom', 'the average income', etc. cannot be excluded by means of the proposed criteria. So, proponents of the neo-Fregean argument should refine their criteria if they wish to insist that singular terms are really distinguishable from other expressions.

To be fair, it should rather be asserted that the already proposed criteria supply necessary but not sufficient conditions for a term to be a singular term. If 't' is a singular term in a sentence then it passes the proposed tests but not all expressions which pass successfully the tests are singular terms. Some of the examples mentioned above, indicate exactly this problem. It should be taken in account that the syntactical distinction between singular terms and other expressions cannot be testified in a satisfactory way, up to now, and that the already proposed criteria need further refinement. Nevertheless, it is a very ambitious task to achieve a complete and sharp distinction between singular terms and other types of expressions and there is no guarantee that even in the future, a set of criteria will be available and adequate enough to characterize, in a definite way, some expressions as singular terms. On the other hand, the Fregean argument about natural numbers as objects is based on the substantival form of arithmetical terms, which is revealed in sentences like "the number of Jupiter's moons is 4" where the arithmetical term behaves like a proper name. Similarly, the sort of argument the neo-Fregean program defends, stresses the thesis that some expressions have to be identified as syntactically functioning as singular terms, before any reference to objects is ascribed to them. Hence, the requirement to fix necessary and sufficient conditions for singular termhood is of great significance. However, this requirement yields a serious difficulty for the substantival account of natural numbers and this difficulty has not yet been coped with.

### 3. Natural numbers as properties

Some philosophers appeal, instead, to the predicative form ' $\exists_n xFx$ ' and argue that numerical quantification is a perfectly well understood notion on its own. (cf. Musgrave, 1986) Numerical definite descriptions like 'the number of Jupiter's moons' could be taken to be quantifiers instead of singular terms. It is also worthwhile to remind here of Hodes' (1984, 123-149) claim that even if numerals are taken to pass the syntactic tests for singular termhood, they not at all manage to effect reference to objects, since their own function is rather to encode cardinality quantifiers than referring to objects.

So, in contrast with Frege, some philosophers hold that there is no reason to reparse sentences like "There are 4 moons of Jupiter" (2) into sentences like "The number of Jupiter's moons is 4" (1). Mill's is one of the old accounts which are closer to the predicative form ' $\exists_n xFx$ ' than to the substantival form ' $Nx:Fx = n$ '. He admitted that arithmetical terms always precede the names of things, for example: 3 pebbles, 2 girls, 10 chairs etc. On this account, numbers can be construed as *properties* of physical things or physical collections. Of course, Frege's attack to

Mill's interpretation is well-known. In *Grundlagen* §§20-23, Frege stressed that numbers cannot be considered as properties of external things in the same way in which e.g. colours can. He argued that an obvious difference between colours and numbers is that a colour characterizes a physical mass in a definite way though the way a number characterizes a physical mass depends upon the *special manner* we choose to consider that physical mass. A crucial point to be stressed here is that Frege's objection to Mill's view is namely that *natural numbers should not be considered independently of some concept F by which we view physical reality*. Indeed, if our concept is *pack of game cards* then we have to apply number 1 but if our concept is *game card* then we have to apply number 52.

Still, however, we can regard natural numbers as properties of concepts, thereby preserving the predicative form of arithmetical expressions. For example the term '4' in the sentence "There are 4 moons of Jupiter" tells us how many instances of the concept *moon of Jupiter* there are. Then each natural number is taken to be a property of a sortal concept.

Arithmetical expressions can perfectly be considered as quantifiers of concepts:

$$\exists_0 x Fx =_{df} \forall x \neg Fx$$

$$\exists_1 x Fx =_{df} \exists x (Fx \wedge \forall y Fy \rightarrow y=x)$$

.....

$$\exists_{n+1} x Fx =_{df} \exists x (Fx \wedge \exists_n y (Fy \wedge y \neq x))^3$$

Accounts of numbers as properties of sets or, as collective properties of many objects together have also been offered by Maddy (1990) and Yi (1999) respectively. Anyhow, regardless of whether we endorse an account of numbers as properties of physical collections or, as properties of concepts, as properties of sets etc., the property-interpretation in general faces traditional difficulties concerning universals. That is, if numbers are construed as properties (universals) then traditional problems concerning properties themselves come up. Are properties to be included in our ontology? How identities between properties are determined? etc. Although numerical identities are perfectly defined in case of quantifiers, metaphysical difficulties arise, concerning the determination of identities between properties. If we decide to construe natural numbers as properties then we will have to deal with property-identities, granted that numbers systematically occur in arithmetical identities. So, we will have to be involved in traditional metaphysical problems concerning ambiguities having to do with determination of property-identities. The question of how identities between properties can be determined remains an issue under discussion among metaphysicians: identities between properties cannot be determined unambiguously since properties may be construed either intensionally or extensionally. Thus, it is possible for two properties to have identical extensions but different intensions.

Ambiguities concerning determination of property-identities are taken under consideration by Maddy (1990, pp. 12-14, 90-94) who prefers to endorse an extensional reading of properties. She holds that although identities between properties can generally be taken to be determined by means of synonymy, this is too strong a condition so it is preferable to treat properties and the identities between them extensionally and in accordance with scientific laws. For example, *being at a*

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<sup>3</sup>. Identity between n and m could be defined as:

$$n = m \text{ iff } \forall F (\exists_n x Fx \leftrightarrow \exists_m x Fx)$$

*temperature of 32 degrees Fahrenheit* and *having such-and-such mean kinetic energy* are not synonymous but they express the same scientific property.

Hence, extensionality is a way out from inconveniences which come up when we have to determine identities between universals. If we insist on a property-interpretation of numbers then we will have to be involved in traditional metaphysical problems concerning determination of identities between properties. However, if we endorse an extensional reading of property-identities, then we shall inevitably return to an object-interpretation. Nevertheless, extensionality leads us back to the object-interpretation, since classes themselves are ordinarily taken to be objects.

However, there is an alternative option. We may still choose the predicative form ' $\exists_n x Fx$ ' but, without ontological commitments to properties at all. Furthermore, we may take the substantival form ' $\exists x: Fx = n$ ' to be reducible to the predicative form ' $\exists_n x Fx$ '. Nevertheless, a reductionist might claim that the kind of things which constitute the reduction basis is simply concrete things and that natural numbers (as abstracta) are eliminable.

To highlight this situation, take e.g. the statement "the number of cats in our house is 2" (3). Then (3) is reducible to the statement "There are 2 cats in our house" (4). Now, the latter can be paraphrased as:

' $\exists x \exists y [Fx \wedge Fy \wedge x \neq y \wedge \forall z (Fz \rightarrow (z=x \vee z=y))]$ '  
(F is the concept *cat in our house*).

This example shows that quantifiers make up a useful instrument for eliminativism. An eliminativist about numbers grants that there are only concrete facts which make arithmetical sentences (3) and (4) true.

One of the main tasks of mathematical antirealism, is nominalization of scientific language, that is, the re-formulation of scientific language in a way such that the alleged mathematical singular terms are eliminated. The very possibility of elimination of the alleged singular terms shows that those terms are not genuine singular terms although their apparent linguistic behaviour implies so. A mathematical nominalist, like Field is an eliminativist so far as he tries to eliminate mathematical terms from the language in which the most important scientific theories are written. To the extent that nominalism keeps on being a viable philosophical program, reduction of the substantival form to the adjectival form offers a useful means for mathematical antirealists to defend their claim that reality is absolutely empty of mathematical entities.

However, nominalism itself has to deal with certain difficulties in order to keep on being a viable philosophical program. For example, nominalists try to divide scientific sentences in a mathematical and a nominalized component and then prove that the nominalized component is the only component which is not eliminable. Yet, Field (1989, p. 235) admits that the division of the mixed scientific language into a mathematical and a nominalized component is not always achieved. He also admits the fact that even when the alleged division is possible, the realistic (mathematical) component often resists elimination. In spite of many efforts being made, nominalization confronts serious difficulties, especially in scientific areas (like quantum mechanics) where mathematics are deeply rooted.

#### **4. The particular-universal distinction**

So far we have taken under consideration two main accounts of natural numbers, a) as objects, an account based on Fregean and neo-Fregean considerations concerning

the linguistic behaviour of arithmetical expressions as singular terms and b) as properties, an account based on the linguistic behaviour of arithmetical expressions as predicates. Sections 2 and 3 aimed to sketch the most serious difficulties each of the two approaches faces. It is interesting that philosophers who take in account linguistic issues in order to support metaphysical claims, choose the one or the other approach, although mathematicians themselves do not distinguish the two forms of syntactical behaviour of arithmetical terms in every day mathematical practice. We've also seen that an attempt to reduce the substantival form ' $Nx:Fx = n$ ' to the predicative form ' $\exists_n xFx$ ' tends to undermine the semantic role of arithmetical terms as genuine singular terms but that there are serious doubts about whether this strategy can further be applied to such an extent that it could be argued that arithmetical terms are really eliminable in general.

The question at issue is about which of the two forms (the substantival or the predicative) represents the metaphysical status of natural numbers and which of the two sentences ("The number of Jupiter's moons is 4" or "There are 4 moons of Jupiter") reveals the correct account of the number 4. This situation, however, reminds us of Ramsey's eccentric claim that there is no real distinction between particulars and universals at least on linguistic grounds. Frank Ramsey (1925) has objected to the common assumption that there is some difference in kind between particulars and universals. He claims that the subject-predicate distinction, even in contexts of atomic sentences, provides no sufficient grounds for a clear distinction between particulars and properties. This is obvious in case of "Socrates is wise" which can be turned round to the equivalent "Wisdom is a characteristic of Socrates". In the latter formulation, wisdom is not a particular although it functions as the subject of the sentence. Ramsey argues that the sentences "Socrates is wise" and "Wisdom is a characteristic of Socrates" assert the same fact and express the same proposition. So, he thinks that which sentence of the two we use is *a matter either of literal style or of the point of view from which we approach the fact*.

Another point Ramsey emphasizes is that the very Fregean distinction between complete and incomplete expressions which are in need of saturation cannot efficiently support any analogous distinction between complete and incomplete entities. Frege's view of an object which is based on the assumption that an expression of it does not contain empty places, can apply as easily to universals as to particulars. Hence, "    is wise" is a predicate because it contains empty places but "wisdom" can perfectly stand for an object since it contains no empty places. Thus, surprisingly, the supposed Fregean conception of an object can apply to universals too. Mellor & Oliver (1997, p. 8) claim that this challenge arises because Frege's account of objects appears to be too broad. So, a point to be stressed here is that even in Frege's sense, *there could be objects which are not particulars*.

If we cannot achieve discrimination between particulars and universals on syntactic grounds then Ramseyan sceptical considerations could be elucidating in case of the dilemma presented in this paper. A possible answer to our dilemma might be that there is no real metaphysical distinction to be asserted on linguistic grounds: which of the two forms (substantival or adjectival) we choose for natural numbers is also *a matter just of literal style or of the point of view from which we approach the (arithmetical) fact*. Nevertheless, such a solution would deflate the metaphysical problem.

## **5. A new option: are the two accounts equivalent?**

In section 3, we considered the option according to which the *substantival* form ‘ $Nx:Fx = n$ ’ is reducible to the predicative form ‘ $\exists_n xFx$ ’ so that the alleged arithmetical singular terms occurring in the first form are eliminable. This is a well-known nominalistic strategy, however, the question which arises is whether the converse reduction strategy could be applied. Why not to take the substantival form as the most fundamental, providing an inflationary reconstruction of familiar numerical quantification? This option reduces the sentence “There are 4 moons of Jupiter” (2) to the sentence “The number of Jupiter’s moons is 4” (1).

In fact, the reduction strategy is not conclusive in either of its two directions. The sentence “The number of Jupiter’s moons is 4” (1) can be considered as a semantic paraphrase of the sentence “There are 4 moons of Jupiter” (2) or vice versa. Nevertheless, semantic paraphrases cannot be conclusive in either direction. They often prove to be useful to both nominalists and realists at their disputes, because realists put forward some sentences ontologically committed to abstracta and nominalists return their nominalistic paraphrases back. (cf. Loux & Zimmerman, 2003, p. 22) Yet, one can interpret paraphrases either in an inflationary or a deflationary way. So, among the two forms in question (the substantival and the predicative form) there is no prevalent.

An alternative option might be to consider the substantival form and the predicative form as equivalent to each other. In this case, we will need to consider principle Nq:  $Nx:Fx = n \leftrightarrow \exists_n xFx$  which is presented by Hale & Wright (2001, pp. 330-332). Nq shows that for each Fregean number  $n$  it is established that  $n$  is the number of the concept  $F$  iff there are exactly  $n$   $F$ s. Nq can be taken as no less than a material equivalence according to which the two sides have the same truth conditions. Independently of the reasons for which neo-Fregeans themselves are interested in Nq, my concern with Nq in this paper is due to the fact that this principle appears to present the substantival and the predicative use of arithmetical expressions as equivalent to each other. It seems that numerical expressions behave equivalently both as singular terms and as predicates of concepts and this is exactly the focal point of this paper.

According to the new option, the two opponent accounts prove to be two sides of the same coin. So, there could be a more metaphysically loaded answer to our dilemma than this suggested in section 4. Numbers can still be taken as (abstract) entities with two different modes of linguistic presentation. The supposed equivalence might have something more to say about the metaphysical status of natural numbers: namely, that numbers are both universals and objects. Can be any universals which are objects? In the previous section, it was noted that some philosophers regard Frege’s view of an object quite broad and it was claimed that even in Frege’s sense, *there could be objects which are not particulars*. So, a first task is to stress that the notion of *object* is distinct from the notion of *particular*. Besides, the assertion that there can be entities which are both objects and universals is endorsed at least by one philosopher, Jonathan Lowe. Lowe (2006, p. 77) accepts entities which are universals but which also satisfy identity conditions, so they are objects as well.<sup>4</sup> Numbers insist on presenting themselves in language in two ways, so, if we allow for the possibility that natural numbers can be both objects and universals then this possibility may justify their double mode of presentation.

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<sup>4</sup> In his opinion, e.g. *kinds* are quite as much objects as particulars are. However, it is not the purpose of this paper to defend Lowe’s account of natural numbers as kinds but only to allow for the possibility that numbers could be both objects and universals.

According to Lowe (2006, p. 75), objects can be discriminated from other entities in virtue of the fact that *objects have determinate identity conditions*. This point is also in agreement with *Grundlagen* (§62) where Frege argues that objects are determined by means of identity statements. As it has already been noted, Hume's principle is an identity criterion for natural numbers since it settles truth conditions for arithmetical identity statements. So, we have to admit that, undoubtedly, natural numbers *are* objects, although they syntactically appear to behave in two different ways. Moreover, we saw that those ways are equivalent. Yet, objects are not necessarily particulars, so the case cannot be excluded that numbers are objects, being also universals.

Let us think of the so called "the paradox of 'the concept horse'". Frege takes 'the concept horse' ('the property of being a horse') to be a singular term while he takes '\_\_\_ is a horse' to be a predicate. So, 'the concept horse' does not co-refer with '\_\_\_ is a horse' because the former stands for an object while the latter stands for a property. The point to be stressed is that there are Fregean objects corresponding to Fregean concepts. The so called paradox should rather be taken to be a situation which provides a double vision in ontology. If we are not willing to endorse a double vision approach then we will keep on considering it as a paradox. For example, in contrast with the Fregean picture, one following her intuitions might grant that both 'wisdom' and '\_\_\_ is wise' refer to the same property. However, the issue here is different, since the relation between a predicate and the correlated property, does not need to be construed as a relation between referring expressions and their referents. We will follow the suggestion made by Hale & Wright (2001, pp. 86-88) that predication is not a species of reference. Relations between predicates and the associated concepts or properties are better characterized by means of *ascription*. It should be granted that a predicate *ascribes* a property or a concept, it does not *refer* to that property or concept. Thus, the property wisdom should be considered as the *ascriptum* of the predicate '\_\_\_ is wise' (not as the referent). Similarly the property or the concept of being a horse should be considered as the *ascriptum* of the predicate '\_\_\_ is a horse'. So, the expressions 'the concept horse' and '\_\_\_ is a horse' differ precisely in that the former *refers to* the concept horse whereas the latter *ascribes* the concept horse.

This proposal has a very elucidating consequence for our dilemma. In this setting, the concept horse *is an object*, as the referent of the Fregean singular term 'the concept horse' *and a concept too*, as the ascriptum of the predicate '\_\_\_ is a horse'. Similarly, wisdom *is an object*, as the referent of the Fregean singular term 'wisdom' and it *is a concept (or a property) too*, as the ascriptum of the predicate '\_\_\_ is wise'.

An account should be endorsed according to which there are also Fregean-type reasons for accepting entities which are both objects and universals. Those entities are objects as the referents of certain Fregean singular terms *and* properties too as the ascripta of predicates. It appears then that the very fact that language offers a double aspect of numbers indicates a double status at the metaphysical level. Though we cannot always achieve strict syntactical distinctions, the role of language should be considered at least as suggestive.

In this paper, it was emphasized that arithmetical expressions behave in two ways in arithmetical language, namely as singular terms and as predicates of concepts. Those forms of syntactical behaviour give rise to metaphysical accounts of natural numbers either as objects or as properties, formulating thereby a dilemma for philosophers of arithmetic. This paper also took under consideration two basic accounts, one regarding each natural number as an object, the other taking each

natural number to be a property of a concept. However, the aim was to show that the very fact of double linguistic presentation should indicate that natural numbers are entities with a double metaphysical status. So, we can now cope with our initial dilemma: which of the two is the number 4? Is it an object? Or is it a property of a concept? The dilemma could be addressed by means of the suggestion that 4 is an abstract entity which is *both an object and a property of a concept too*. It is an object for which a corresponding singular term stands. It is also a property which a corresponding predicate ascribes. Yet in this case, the two forms of linguistic behaviour of arithmetical expressions should be regarded as indicative of the Janus' metaphysical face of natural numbers.

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